Roll No.

(12/14-I)

3151

M. Sc. (5 Years) EXAMINATION

(For Batch 2010 & Onwards)

(First Semester)

MATHEMATICS

MHT-1101

Algebra

Time : Three Hours *Maximum Marks* : 40

Note : Section A (Compulsory Section)—Attempt all the questions.

Section B—Attempt any *five* questions out of eight.

Section C—Attempt any *two* questions out of four.

Section A

(Compulsory)

1. Find the characteristic roots of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}.$$ 2

2. Remove the second term from the equation :

$$x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$$ 2

3. Prove that every skew matrix of odd order is a singular matrix. 2

4. Apply Descarte's rule of signs to discuss the nature of the roots of equation : 2

$$x^4 + 15x^2 + 7x - 11 = 0$$

Section B

5. If 1, α, β, γ,........ are n roots of unity, prove that :

$$(1-\alpha)(1-\beta)(1-\gamma)\text{..........} = n$$ 4

6. Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ to $[I_3, 0]$. Hence find $f(A)$. 4

7. State and prove Cayley's-Hamilton theorem. 4

8. Diagnolize the quadratic form : 4

$$x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$$

Also find the equation of transformation.

9. Solve the equation $x^3 - 12x - 65 = 0$ by Cardon's method. 4

10. Find the equation of squared difference of the roots of the equation $x^3 + 3x - 2 = 0$. 4

11. Show that if A is Hermitian and P is unitary, then $P^{-1}AP$ is Hermitian. 4

12. Prove that the set of vectors $u = (1, 3, 2)$, $v = (1, -7, -8)$, $w = (2, 1, -1)$ is linearly dependent. 4

Section C

13. Apply Descarte's method to solve the equation : 6

$$2x^4 + 6x^3 - 3x^2 + 2 = 0$$

14. Solve the equation :

$$x^4 - 7x^3 + 18x^2 - 20x + 8 = 0,$$

which has multiple roots. 6

15. For what value of K, the equations : 6

$$x + y + z = 1$$
$$x + 2y + 4z = K$$
$$x + 4y + 10z = K^2$$

have a solution and solve them completely in each case.

16. Prove that the rank of the tranpose of a matrix is the same as that of the original matrix. 6